Department of Mathematics
University of Notre Dame

David Galvin - University of Notre Dame

GRADUATE STUDENT SEMINAR

Guest Speaker: David Galvin

University of Notre Dame

Date: Monday, October 1, 2018

Time: 4:00 PM

Location: 231 Hayes-Healy Hall

Lecture Title:

Total non-negativity of some combinatorial matrices

Abstract

Many combinatorial matrices — such as those of binomial coefficients, Stirling numbers of both kinds, and Lah numbers — are known to be totally non-negative, meaning that all minors (determinants of square submatrices) are non-negative. The examples noted above can be placed in a common framework: for each one there is a non-decreasing sequence $(a_1, a_2, \ldots)$, and a sequence $(e_1, e_2, \ldots)$, such that the (m, k)-entry of the matrix is the coefficient of the polynomial $(x-a_1)\cdots(x-a_k)$ in the expansion of $(x-e_1)\cdots(x-e_m)$ as a linear combination of the polynomials $1, x-a_1, \ldots, (x-a_1)\cdots(x-a_m)$. I'll discuss this general framework, and for a non-decreasing sequence $(a_1, a_2, \ldots)$ sketch the proof of necessary and sufficient conditions on the sequence $(e_1, e_2, \ldots)$ for the corresponding matrix to be totally non-negative. I'll derive as corollaries the totally non-negativity of matrices of rook numbers of Ferrers boards, and of a family of matrices associated with chordal graphs.