University of Notre Dame Department of Mathematics

COLLOQUIUM

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Will give a lecture entitled:

Grauert tubes in *TM* and some applications

On

Wednesday, August 25, 2010

At

4:00 PM

In

117 Hayes-Healy Hall

Abstract

For a real-analytic Riemannian manifold (M, g), there exists a unique complex structure on an open neighborhood U of M in TM which has turned every geodesic $\gamma \subset M$ into a holomorphic curve $\gamma^{\mathbb{C}} \subset U$.

The disc bundle of radius r equipped with this complex structure is called a *Grauert tube* T^rM *over* M *of radius* r.

• In the first part of the talk, we will talk about some rigidity result and one of its applications.

Theorem I: T^rM is the ball if and only if M is the real hyperbolic space and $r = \pi/2$.

Theorem II: Let M be a homogeneous space. Then either T^rM is covered by the ball or $Aut(T^rM) = Isom(M)$.

Theorem III: Let G be a connected Lie group of dimension $n \geq 2$. Then there exists a complete hyperbolic Stein manifold $\Omega, dim_{\mathbb{C}}\Omega = n$, such that $Aut(\Omega) = G$.

• The second part of the talk will concentrate on a rescaling method to obtain a complete Ricci-flat metric on TM when M is a compact symmetric space of rank-one.

The first observation is that the Kähler-Einstein metric on T^rM of scalar curvature -1 has a Kähler potential satisfying the ODE:

$$h''(u)(h'(u))^{(n-1)}\exp(-(n+1)h(u)) = u^{n-1}\hat{S}(u).$$

Taking a positive decreasing sequence $\{\lambda_r\}_{r>0}, \lim_{r\to\infty}\lambda_r = 0$, we rescale the metric so that g_r is the complete Kähler-Einstein metric in T^rM of Ricci curvature $-\lambda_r$. The idea is to show the limiting metric $\lim_{r\to\infty} g_r$ is a Ricci-flat metric in TX.