

**MATH 80750 FALL 2018**  
**TOPICS IN DIFFERENTIAL GEOMETRY:**  
**EINSTEIN MANIFOLDS**

INSTRUCTOR: Matt Gursky

This course will be a survey of various topics related to the existence problem for (Riemannian) Einstein metrics.

- **ANALYTIC PRELIMINARIES:** Some basic tools from functional analysis and PDEs; Sobolev spaces and elliptic regularity.
- **SURFACES:** The Uniformization Theorem.
- **CURVATURE IN HIGHER DIMENSIONS:** The curvature tensor; decompositions. The Weyl, Ricci, and Schouten tensors. The scalar curvature. Linearization formulas.
- **VARIATIONAL ASPECTS:** The total scalar curvature functional. First and second variations; splitting theorems for tensor fields; conformal versus transverse-traceless variations.
- **QUADRATIC CURVATURE FUNCTIONALS:** Four dimensions; the Bach tensor. Self-duality.
- **THE YAMABE PROBLEM:** The Yamabe problem: the case of non-positive scalar curvature. The Positive Mass Theorem.
- **THE SINGULAR YAMABE PROBLEM:** The work of Loewner-Nirenberg.
- **POINCARÉ-EINSTEIN METRICS:** Fefferman-Graham expansions; the renormalized total scalar curvature.
- **EXISTENCE PROBLEM FOR POINCARÉ-EINSTEIN METRICS:** Graham-Lee perturbation result; elliptic theory and weighted spaces. Rigidity and non-existence results.
- **MODULI SPACE OF EINSTEIN METRICS:** Rigidity. The set of Einstein constants.