Topology Seminar

UNIVERSITY OF NOTRE DAME
Department of Mathematics

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2:30 PM
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Title: A modified Bott conjecture in cohomogeneity two

Abstract:

A central problem in Riemannian geometry is to find obstructions to positive and non-negative curvature. To this end a fundamental conjecture attributed to Bott suggests: "The Betti numbers of the loop space of a positively/non-negatively curved manifold, M grow at most polynomially" When restricted to the field, Q of rational numbers, this property (for closed simply connected manifolds) is equivalent to M having finite dimensional rational homotopy, i.e., all but finitely many homotopy groups of M are finite. This is a severely restrictive property referred to as being rationally elliptic. Examples of rationally elliptic manifold, M include closed simply connected homogeneous manifolds M = G/H as well as those of cohomogeneity one, i.e., M admits a G action with orbit space M/G of dimension one. In contrast, the vast majority of cohomogeneity two manifolds are not rationally elliptic. On the geometric side, it is a basic fact that all closed homogeneous manifolds M also have a(n invariant) metric of non-negative curvature. This, however, is false in cohomogeneity one, but they do have metrics of almost non-negative curvature We will explain that Closed 1-connected cohomogeneity two manifolds M of almost non-negative curvature are rationally elliptic. This is joint work with B. Wilking and J. Yeager. All notions will be explained.