Cluster Algebras Seminar

UNIVERSITY OF NOTRE DAME
Department of Mathematics

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4:30 PM
258 Hurley Hall

Title: Gelfand-Zeitlin integrable systems: where linear algebra, geometry, and representation theory meet

Abstract:

In the 19th century, physicists were interested in determining the conditions under which the equations of motion for a classical mechanical system could be found by integrating a finite number of times. Such a system was said to be completely integrable. Using symplectic geometry, we can generalize the notion of an integrable system beyond the realm of physics and into Lie theory and representation theory. Such “abstract” integrable systems can be used to geometrically construct infinite dimensional representations of Lie algebras. In this talk, I will discuss a family of integrable systems, the Gelfand-Zeitlin systems, that arise from purely linear algebraic data. For an n × n complex matrix X, we consider the eigenvalues of all the i × i submatrices in the top left hand corner of X. These are known as Ritz values and play an important role in numerical linear algebra. We will see how questions about Ritz values naturally lead to the construction of the Gelfand-Zeitlin integrable systems. I will explain results about the geometric properties of these systems and indicate how they answer questions of Parlett and Strang about Ritz values. I will also show how this research provides the foundation for the geometric construction of a category of infinite dimensional representations of certain classical Lie algberas using the theory of quantization.