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258 Hurley Hall

Title: A Liouville type theorem in sub-Riemannian geometry, and applications to several complex variables

Abstract:

The Riemann mapping theorem tells us that any simply connected planar domain is conformally equivalent to the disk. This provides a classification of simply connected domains via conformal maps. This classification fails in higher dimensional complex spaces, as already Poincare' had proved that bi-discs are not bi-holomorphic to the ball. Since then, mathematicians have been looking for criteria that would allow to tell whether two domains are bi-holomorphic equivalent. In the early 70's, after a celebrated result by Moser and Chern, the question was reduced to showing that any bi-holomorphism between smooth, strictly pseudo-convex domains extends smoothly to the boundary. This was established by Fefferman, in a 1974 landmark paper. Since then, Fefferman's result has been extended and simplified in a number of ways. About 10 years, ago Michael Cowling conjectured that one could prove the smoothness of the extension by using minimal regularity hypothesis, through an argument resting on ideas from the study of quasiconformal maps. In its simplest form, the proposed proof is articulated in two steps: (1) prove that any bi-holomorphism between smooth, strictly pseudoconvex domains extends to a homeomorphisms between the boundaries that is 1-quasiconformal with respect to the sub-Riemannian metric associated to the Levi form; (2) prove a Liouville type theorem, i.e. any $1-$quasiconformal homeomorphism between such boundaries is a smooth diffeomorphism. In this talk I will discuss recent work with Le Donne, where we prove the first step of this program, as well as joint work with Citti, Le Donne and Ottazzi, where we settle the second step, thus concluding the proof of Cowling's conjecture. The proofs draw from several fields of mathematics, including nonlinear partial differential equations, and analysis in metric spaces.