Cluster Algebras Seminar

UNIVERSITY OF NOTRE DAME
Department of Mathematics

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4:00 PM
258 Hurley Hall

Title: Introduction to thick morphisms: from higher Koszul brackets to BV operators

Abstract:

Many mathematical structures such as even and odd Poisson brackets, Lie algebroids and their generalizations, L-infinity algebras... can be described by binary or higher derived brackets generated by some "master" homological element, e.g. a homological vector field or an odd master Hamiltonian. The natural language for that is provided by supergeometry. In the talk, I will introduce thick morphisms of supermanifolds, which generalize ordinary smooth maps and induce non-linear, in general, pullbacks on functions. This nonlinearity is crucial for having L-infinity morphisms of homotopy brackets. My original motivation was the case of "higher Koszul brackets" of differential forms on a homotopy Poisson manifold. With the help of thick morphisms, we construct an L-infinity morphism of forms to multivectors taken with the canonical Schouten bracket. This example is model for a general setting of L-infinity (bi)algebroids. There are two parallel theories of thick morphisms: bosonic and fermionic (acting on even and odd functions, respectively). As it turns out, bosonic thick morphisms can be understood as classical limits of certain "quantum thick morphisms", which are Fourier integral operators of a particular kind. (It is interesting that these are close to the operators introduced in 1950s-1960s independently by V. Fock in physics, following Dirac, and by Vishik, Eskin, Egorov, Fedoryuk in the theory of PDEs, and which preceded and motivated Hörmander's general notion.) We show how the connection with homotopy S-infinity structures is lifted to the level of "quantum Batalin-Vilkovisky algebras".