**Speaker:** David Galvin
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Thursday, September 14, 2017
2:00 PM
258 Hurley Hall

**Title:** Plates, olives, and Morse theory

**Abstract:**

Morse theory seeks to understand the shape of a manifold by studying smooth functions on it. Although a smooth function on a smooth manifold is an inherently continuous object, the study of Morse functions can lead to interesting and non-trivial combinatorial questions. For example, in 2006 Arnold asked how many Morse functions the sphere admits. Fixing the dimension of the sphere, the number of critical points of the function and some natural notion of equivalence, this becomes a discrete question, and has connections to well-known combinatorial objects such as alternating permutations, Catalan paths and Young's lattice. One version of the problem leads to Nicolaescu's *game of plates and olives*. Start with an empty table, and at each step either add an empty plate, or add an olive to a plate, or eat an olive from a plate, or remove an empty plate, or combine the olives from two plates and remove one of the plates. The number of games of length $2n$ that start and end with an empty table is (essentially) the number of Morse functions on $S^2$ with $n$ saddle points, up to a notion of topological equivalence. We have identified the growth rate of this quantity, answering a question of Nicolaescu. It's an area where plenty more work remains to be done, though. Joint work with Teena Carroll, Emory & Henry College.