

## FELIX KLEIN SEMINAR

**Guest Speaker: Fuquan Fang**  
**Capital Normal University, Beijing**

**Date:** Thursday, August 31, 2017

**Time:** 2:00 PM

**Location:** 258 Hurley Hall



**Lecture Title:**

**Dual submanifolds in rational homology spheres**

**Abstract**

Let  $\Sigma$  be a rational homology sphere. A pair of disjoint closed submanifolds  $M_+, M_- \subset \Sigma$  are called *dual* to each other if the complements  $\Sigma - M_+$  strongly homotopy retracts onto  $M_-$  or vice-versa. In this paper we are concerned with the basic problem of which integral triples  $(n; m_+, m_-) \in \mathbb{N}^3$  can appear, where  $n = \dim \Sigma - 1$ ,  $m_{\pm} = \text{codim} M_{\pm} - 1$ . The problem is motivated by several fundamental aspects in differential geometry. Our main result provides a surprising simple answer, namely, if and only if one of the following holds true:  $m_+ = m_- = n$ ,  $m_+ = m_- = \frac{1}{3}n \in \{1, 2, 4, 8\}$ ,  $m_+ = m_- = \frac{1}{6}n \in \{1, 2\}$ ,  $\frac{n}{m_+ + m_-} \in \{1, 2\}$ . Moreover, assuming  $\frac{n}{m_+ + m_-} = 2$ , if  $m_+ = m_-$ , then  $m_+ = m_- \in \{1, 2\}$ ; if  $m_+ > m_- \geq 2$ , then  $m_+ + m_-$  is odd.