Topology Seminar

UNIVERSITY OF NOTRE DAME
Department of Mathematics

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Tuesday, April 4, 2017
2:30 PM
258 Hurley Hall

Title: 4-dimensional analogues of Dehn’s lemma

Abstract:

Dehn's lemma is a classical and fundamental result for 3-manifolds. We investigate certain 4-dimensional analogues, giving examples where they do or do not hold, in the smooth and topological categories. For instance, we show that an essential 2-sphere S in the boundary of a simply connected 4-manifold W such that S is null-homotopic in W need not extend to an embedding of a ball in W. However, if W is simply connected (or more generally, has abelian fundamental group) with boundary a homology sphere, then S bounds a topologically embedded ball in W. Moreover, we give examples where such an S does not bound any smoothly embedded ball in W. We give similar results for tori; in particular, we construct an incompressible torus T in the boundary of a contractible 4-manifold W such that T extends to a topological embedding of a solid torus in W but no smooth embedding. (This is joint work with Danny Ruberman.)