

## **ALGEBRAIC GEOMETRY AND COMMUTATIVE ALGEBRA SEMINAR**

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**Date:** Friday, February 17, 2017

**Time:** 4:00 PM

**Location:** 258 Hurley Hall

**Lecture Title:**

**Finite free resolutions and Kac-Moody Lie algebras**

### **Abstract**

Let us recall that a format  $(r_n, \dots, r_1)$  of a free complex  $0 \rightarrow F_n \rightarrow F_{n-1} \rightarrow \dots \rightarrow F_0$  over a commutative Noetherian ring is the sequence of ranks  $r_i$  of the  $i$ -th differential  $d_i$ . We will assume that  $\text{rank } F_i = r_i + r_{i+1}$ . We say that an acyclic complex  $F_{\text{gen}}$  of a given format over a given ring  $R_{\text{gen}}$  is generic if for every complex  $G$  of this format over a Noetherian ring  $S$  there exists a homomorphism  $f: R_{\text{gen}} \rightarrow S$  such that  $G = F_{\text{gen}} \otimes_{R_{\text{gen}}} S$ . For complexes of length 2 the existence of the generic acyclic complex was established by Hochster and Huneke in the 1980's. It is a normalization of the ring giving a generic complex (two matrices with composition zero and rank conditions). I will discuss the ideas going into the proof of the following result: Associate to a triple of ranks  $(r_3, r_2, r_1)$  a triple  $(p, q, r) = (r_3 + 1, r_2 - 1, r_1 + 1)$ . Associate to  $(p, q, r)$  the graph  $T_{p,q,r}$  (three arms of lengths  $p - 1, q - 1, r - 1$  attached to the central vertex). Then there exists a Noetherian generic ring for this format if and only if  $T_{p,q,r}$  is a Dynkin graph. In other cases one can construct in a uniform way a non-Noetherian generic ring, which deforms to a ring carrying an action of the Kac-Moody Lie algebra corresponding to the graph  $T_{p,q,r}$ .