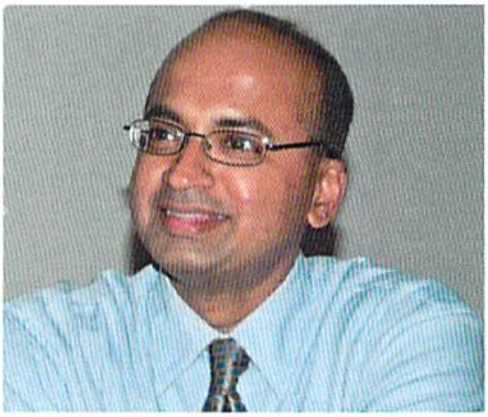
Colloquium

University of Notre Dame
Department of Mathematics

Speaker: Krishnan Shankar

University of Oklahoma



Will give a lecture entitled

New metrics of non-negative sectional curvature on exotic spheres

Date: Wednesday, October 5, 2016

Time: 4:00 PM

Location: 117 Hayes-Healy Hall

Departmental Tea: Tea in Room 257 (lounge in Hurley Hall) at 3:30 p.

Abstract:

A manifold of dimension $2n+1$ is said to be *highly connected* if its first $(n-1)$ homotopy groups are trivial i.e., $\pi_i(M) = 0$ for all $i \leqslant n-1$. It was the study of highly connected, 7-manifolds that led Milnor to his seminal discovery of manifolds that are homeomorphic, but not diffeomorphic, to the standard sphere $\mathbf{S}^7$. It was shown by Milnor and others that there are 28 distinct oriented, diffeomorphism types of such *exotic spheres*. The celebrated Sphere Theorem of Rauch states that a positively curved manifold M^n whose sectional curvatures are strictly $\frac{1}{4}$-pinched, i.e., $\dfrac{\min\, textbf\, sec_M}{\max\, \mathbf{sec}_M} > \dfrac{1}{4}$, must be homeomorphic to a sphere. Which naturally begs the question: do exotic spheres admit metrics of positive or even non-negative sectional curvature?

In 1974, D. Gromoll and W. Meyer exhibited a single exotic sphere in dimension 7 as a biquotient, i.e., a double coset manifold and therefore admits a metric of non-negative sectional curvature. Until 2000 this was the only known example when K. Grove and W. Ziller constructed such metrics on all the so called *Milnor spheres* in dimension 7; these are exotic spheres which are additionally $\mathbf{S}^3$-bundles over $\mathbf{S}^4$. This left 8 out of 28 exotic spheres in dimension 7 which do not admit such a bundle structure nor were they known to admit non-negative sectional curvature. In this talk we present a new method of construction which builds on the Grove–Ziller method and yields non-negative curvature on all 28 spheres in dimension 7. Additionally it produces many new examples of highly connected, 7-manifolds with non-negative sectional curvature beyond the collection of exotic spheres. This is joint work with Sebastian Goette and Martin Kerin.

Krishnan Shankar - University of Oklahoma