Cluster Algebras Seminar

UNIVERSITY OF NOTRE DAME
Department of Mathematics

Speaker: Arlo Caine
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Monday, September 19, 2016
4:00 PM
258 Hurley Hall

Title: Cohomology of Toric Poisson Structures on $\mathbb{C}^n$

Abstract:

The Poisson cohomology of a Poisson manifold (M, π) is the cohomology of the differential graded algebra composed of the exterior algebra of $\mathcal{V}(M)$ of all multi-vector fields on M, equipped with the differential $\sigma(X) = [X, \pi]$, where $[\cdot, \cdot]$ is the Schouten bracket of multi-vector fields. It is notoriously difficult to compute, and few non-trivial examples are understood. We consider complex algebraic Poisson structures on $\mathbb{C}^n$ which are invariant under the action of the complex torus $(\mathbb{C}^\times)^n$. Such structure is determined by a skew-symmetric matrix of complex numbers. The Lie algebra of the torus is included as an abelian Lie subalgebra $\mathfrak{g}$ of $\mathcal{V}^1(\mathbb{C}^n)$ and its exterior algebra $\bigwedge \mathfrak{g}$ is a subalgebra of $\mathcal{V}(\mathbb{C}^n)$. We prove that the cohomology of such a Poisson structure is a module over $\bigwedge \mathfrak{g}$ and that it is generated by monomial decomposable multi-vectors corresponding to the solutions of Diophantine inequalities determined by A. Consequently, we obtain a vast new class of non-trivial examples in which the Poisson cohomology can be computed. This is joint work with Berit Givens of California State Polytechnic University, Pomona.