Felix Klein Seminar

UNIVERSITY OF NOTRE DAME
Department of Mathematics

Speaker: **Frederick Wilhelm**
University of California, Riverside

Thursday, April 14, 2016
2:00 pm
Room: 125 Hayes-Healey Hall

Title: Focal Radius, Rigidity, and Lower Curvature Bounds

Abstract:
We show that the focal radius of any submanifold N of positive dimension in a manifold M with sectional curvature greater than or equal to 1 does not exceed $\frac{\pi}{2}$. In the case of equality, we show that N is totally geodesic in M and the universal cover of M is isometric to a sphere or a projective space with their standard metrics, provided N is closed. Our results also hold for k^{th}–intermediate Ricci curvature, provided the submanifold has dimension $\geq k$. Thus in a manifold with Ricci curvature $\geq n-1$, , Riemannian submersions, and submetries of positively curved manifolds, including generalizations of some of the sectional curvature results of Chen and Grove. (Joint work with Luis Guijarro.) To prove the results, we use a new comparison lemma for Jacobi fields that exploits Wilking's transverse Jacobi equation. The new comparison lemma also yields an optimal estimate for the norm of any submanifold's second fundamental form in terms of its focal radius and the ambient manifold's lower curvature bound. This leads to a 'soul-type' structure theorem for manifolds with nonnegative k^{th}–intermediate Ricci curvature that have a closed submanifold with dimension $\geq k$ and infinite focal radius.