Logic Seminar

UNIVERSITY OF NOTRE DAME
Department of Mathematics

Speaker: Joseph Miller
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Tuesday, February 23, 2016
2:00 PM
125 Hayes-Healy Hall

Title: Generic Muchnik reducibility and expansions of the reals

Abstract:

If A and B are countable structures, then A is Muchnik reducible to B if every ω-copy of B computes an ω-copy of A. This can be interpreted as saying that B is intrinsically as complicated as A. While this is a natural way to compare the complexity of structures, it was limited to the countable setting until Schweber introduced an extension to arbitrary structures: if A and B are (possibly uncountable) structures, then A is generically Muchnik reducible to B if in some (equivalently, any) forcing extension that makes A and B countable, A is Muchnik reducible to B. I will discuss what we know about generic Muchnik reducibility as it pertains to expansions of Cantor space, Baire space, and the real numbers. For example, we will show that Baire space is generic Muchnik equivalent to any expansion of the reals by countably many continuous functions. On the other hand, if we expand Cantor space by adding jump and join, then we get a strictly more complicated structure, one that is generic Muchnik above every Borel structure. These results are proved by translating them to facts about the complexity of listing the sets in countable Turing ideals. For example, from a listing of the infinite sets in a countable jump ideal, it is possible to compute a listing of the sets along with the jumps of joins of members of the list. On the other hand, it is not always possible to compute a listing along with jump and join as functions on indices. This work is joint with Andrews, Knight, Kuyper, Lempp, and Soskova. It builds on work of Knight, Montalbán, and Schweber; Igusa and Knight; Igusa, Knight, and Schweber; and Downey, Greenberg, and myself.