Speaker: Arkady Berenstein
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Friday, March 18, 2016
4:00 PM
117 Hayes-Healy Hall

Title: Hecke-Hopf algebras

Abstract:

Hecke algebras H_q(W) of Coxeter groups W first emerged in the study of Chevalley groups in mid sixties and since then became central objects in Representation Theory of Coxteter groups and semisimple Lie groups over finite fields. In particular, as a one-parameter deformation of the group algebra kW of W, the Hecke algebra H_q(W) helps to classify representations of W and to equip each simple kW-module with the canonical Kazhdan-Lusztig basis. Unfortunately, unlike the group algebra kW, the Hecke algebra H_q(W) lacks a Hopf algebra structure, that is, it is not clear how to tensor multiply H_q(W)-modules. Moreover, there is a general consensus that a naive Hopf structure on H_q(W), if exists, would essentially coincide with that on kW, so we would not gain any new information. In may talk (based on joint work with D. Kazhdan) I suggest a roundabout: instead of forcing a naive Hopf structure on H_q(W), we find a ``reasonably small'' Hopf algebra H(W) (we call it Hecke-Hopf algebra of W) that "naturally" contains H_q(W) as a coideal subalgebra. The immediate benefit of this enlargement of H_q(W) is that each representation of H(W) and each representation of H_q(W) can be tensor multiplied into a new representation of H_q(W), thus allowing to create infinitely many new H_q(W)-modules out of a single one. Hecke-Hopf algebras have some other applications, most spectacular of which is the construction of new infinite families of solutions to the quantum Yang-Baxter equation.