Colloquium

UNIVERSITY OF NOTRE DAME
Department of Mathematics

Speaker: Brent Doran
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Thursday, January 28, 2016
4:00 PM
125 Hayes-Healy Hall

Title: Variations on a theme in G: algebraic group actions in unexpected places in geometry, arithmetic, and physics

Abstract:

Let G be a connected algebraic group; then G is "built from" copies of G_a, the additive group, and G_m, the multiplicative group. Multiplicative group actions are well-understood, in a sense are finite or discrete in nature, and have fixed point behavior that determines much of their structure. Additive components to group actions have been much harder to understand. Nevertheless, when "tamed" by the presence of a compatible multiplicative action, they can be studied in detail, following my work with Kirwan and others. Such actions are everywhere, once you start to look, and are often tightly tied to familiar yet challenging geometry. As an application of this program, one can take a "bird's eye view" on a number of seemingly disparate well-known problems: e.g., fitting curves of minimal degree through points in the plane, effectivity of homology classes in certain varieties, counting rational points of bounded height in Fano varieties over number fields, birational geometry of moduli of curves, hypergeometric differential equations and periods of complete intersections, and even quantum entanglement and questions in computational complexity theory. From this perch, many mathematical structures just beyond the our training and comfort zone -- like non-linearizable actions, A^1-bundles that are not line bundles, non-finitely generated rings, infinite strings of wall-crossings -- become interelated, different sides of the same die. More importantly, they become accessible to study, providing a route to important modern conjectures in a way that resonates with the algebraic geometry's roots in the late 19th century.