



Speaker: Harold Williams
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Thursday, September 17, 2015

2:00 PM

125 Hayes-Healy Hall

Title: Cluster Combinatorics of Symplectic 4-Manifolds and Legendrian Knots

Abstract:

We explore a relationship between combinatorics and symplectic geometry, in particular the geometry of certain noncompact 4-manifolds (for example, cotangent bundles of surfaces) and their contact boundaries (for example, cosphere bundles of surfaces). The combinatorics comes from the theory of cluster algebras, a kind of unified theory of canonical bases in representation theory and algebraic geometry. Some basic features of cluster algebras are that they are defined from purely combinatorial data (for example, a quiver) and they are coordinate rings of varieties covered by algebraic tori with transition functions of a special, universal form. Despite the originally representation-theoretic motivation for the subject, connections between cluster theory and symplectic geometry emerged later through the appearance of cluster-theoretic formulae in wall-crossing phenomena and mirror symmetry. We will discuss recent work expanding on this connection, in particular providing a universal framework for interpreting cluster varieties as moduli spaces of objects in Fukaya categories of Weinstein 4-manifolds. This connects the combinatorics of cluster algebras to, for example, the classification of exact Lagrangian fillings of Legendrian knots. An accompanying theme, which plays a key role both technically and in relating the symplectic picture of cluster algebras to more standard representation-theoretic ones, is the role of categories of microlocal sheaves as topological counterparts of Fukaya categories. This is joint work with Vivek Shende, David Treumann, and Eric Zaslow.