

# Computability Seminar



**Speaker:** Rose Weisshaar  
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Thursday, September 10, 2015  
3:00 pm  
Room: 125 Hayes-Healy Hall

**Title:** Understanding the Paris-Harrington Theorem, Part I

**Abstract:**

Call a finite set of natural numbers  $X$  *relatively large* if  $|X| \geq \min(X)$ . Given natural numbers  $M$ ,  $e$ ,  $r$ , and  $k$ , let  $M \overset{*}{\rightarrow} (k)_r^e$  denote the statement that for every partition  $P : [M]^e \rightarrow r$  of the size  $e$  subsets of  $M$  into  $r$  colors, there is a relatively large set  $X \subseteq M$  that is homogeneous for  $P$ . It is not difficult to show, using the infinite Ramsey theorem and a compactness argument, that

$$(*) \quad (\forall e, r, k)(\exists M)(M \overset{*}{\rightarrow} (k)_r^e).$$

However, Paris and Harrington<sup>1</sup> showed that the statement  $(*)$  cannot be proved in Peano arithmetic. We will try to present a clear explanation of their proof.

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<sup>1</sup>A Mathematical Incompleteness in Peano Arithmetic, Handbook of Mathematical Logic, 1977