Math 80510, Topics in Mathematical Logic
Gregory Igusa, Fall 2015
Forcing in Set Theory and Recursion Theory

Introduction: In this course, we define and build up forcing in both set theory and recursion theory side-by-side.

In set theory, forcing is a process by which one takes a model, M, of set theory and expands it by adding sets to create a "forcing extension," $M[G]$: a new model with potentially different properties. The primary purpose of this is as a technique for proving that various statements are consistent with ZFC. The strength of forcing comes from the fact that the new sets are added in a manner that is so controlled that even M is able to discuss what will be true in $M[G]$, allowing $M[G]$ to inherit much of its set-theoretic structure from M.

The notation of forcing creates a paradigm in which many of the "proofs by direct construction" of traditional recursion theory can be expressed more neatly and clearly, due to the manner in which forcing allows one to discuss the properties of a set "before" it is even built. A full understanding of the formalism of forcing is helpful in research, in that it provides a short and easy way to frame a discussion of potential constructions, and also to dismiss approaches that cannot possibly be made to work.

In this course, we will define forcing in both set theory and recursion theory, explain the formalism of how forcing can be used to build models of ZFC, force to prove several consistency results over ZFC, rephrase several results of classical recursion theory in the language of forcing, and later use forcing in recursion theory to prove several more subtle results of recursion theory.

Prerequisites: This course will assume a graduate level understanding of computability theory (Math 60510 or the equivalent) as well as a basic understanding of set theory and possibly model theory. (The basics of forcing in set theory require very little classical set theory to understand and use.)

Homework: Homework will periodically be assigned and corrected.