Topology Seminar

UNIVERSITY OF NOTRE DAME
Department of Mathematics

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Wednesday, April 29, 2015
3:45 pm
Room: 229 Hayes-Healy Hall

Title: Analyzing the Grothendieck ring of varieties using K-theory

Abstract:
The Grothendieck ring of varieties, $K_0[\mathcal{V}_k]$ was introduced by Grothendieck in a 1964 letter to Serre. It is defined to be the free abelian group generated by varieties (over a fixed field k), under the relation that if Y is a closed subvariety of X then $[X] = [Y] + [X \backslash Y]$. Multiplication is defined via the Cartesian product. Thus, in particular, if X and Y are varieties which are piecewise isomorphic then $[X] = [Y]$ in the ring. There are two important structural questions about this ring: 1. If two varieties X and Y satisfy $[X] = [Y]$, are they piecewise isomorphic? 2. Is the class of the affine line a zero divisor? The answers to these questions are "no" and "yes", respectively; this was shown in a recent paper of Borisov, where he constructed an example to resolve question (2), which also coincidentally answered question (1).

In this talk we will analyze the structure of this ring using techniques of algebraic K-theory. In particular, we will construct a spectrum $K(\mathcal{V}_k)$ whose π_0 is the Grothendieck ring of varieties, and whose higher homotopy groups contain further geometric information about piecewise isomorphisms of varieties. We will then analyze the geometry of this spectrum to show that the kernel of multiplication by the affine line is generated by varieties X and Y such that $[X] = [Y]$ but X and Y are not piecewise isomorphic, thus showing that Borisov's coincidence is not a coincidence at all.