

Speaker: Charlie McCoy, CSC
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Tuesday, December 16, 2014
2:00 pm
Room: 125 Hayes-Healey Hall

Title: An effective version of the uniqueness of prime models

Abstract:

A well-known result in classical model theory is that a prime model of a complete, countable theory is unique, up to isomorphism. The standard, classical proof invokes the fact that prime models are atomic and then uses atomic formulas to construct a back-and-forth isomorphism. However, this approach seems non-effective. Indeed, Hirschfeldt, Shore, and Slaman shows that the uniqueness (up to isomorphism) of atomic models is equivalent to ACA_0 , so that there are two decidable atomic models for which any isomorphism between them computes $0'$. Thus, any attempt to analyze the uniqueness of prime models from the perspective of effective model theory or reverse mathematics must deal directly with the fact that the models are prime, and not simply that they are atomic. In work with Cholak, we prove the following:

Theorem Let T be a decidable, complete theory with decidable models \mathcal{A}, \mathcal{B} . Then there is a decidable model $\mathcal{M} \models T$ so that either there is no computable elementary embedding of \mathcal{A} into \mathcal{M} , or there is no computable elementary embedding of \mathcal{B} into \mathcal{M} , or there is a computable isomorphism $h : \mathcal{A} \cong \mathcal{B}$.

This result in recursive model theory has implications in regard to the reverse mathematical strength of the classical theorem.