Speaker: **Noah Schweber**
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Thursday, November 13, 2014
3:00 pm
Room: 125 Hayes-Healey Hall

Title: Computability of uncountable structures

Abstract:
In computable structure theory, we seek to understand the complexity of a structure (or relations on a structure) by looking at its copies, that is, isomorphic structures with domain ω. This approach is of course limited to *countable* structures. It is natural to ask to what extent classical computable structure theory can be "lifted" to uncountable structures, without changing the picture in the countable setting.

In this talk, I'll show how this can be achieved by looking at the behavior of uncountable structures in extensions of the set-theoretic universe in which they are made countable. In particular, I will focus on the "generic" version of a natural computability-theoretic preorder on countable structures, *Muchnik reducibility*: A is Muchnik reducible to B if every copy of B computes a copy of A. I will prove basic properties of this reducibility, and present a few examples:

- the field of complex numbers is generically computably presentable; - if a, b are ordinals and $a < b$, then a is generically Muchnik reducible to b; - every countable structure is generically Muchnik reducible to the field of real numbers; and - a countable structure A is generically Muchnik reducible to the ordinal ω_1 if and only if A is generically Muchnik reducible to some countable ordinal.

I will also sketch a recent proof, due to Julia Knight and Greg Igusa, that the field of real numbers is strictly more complex than the powerset of ω. Finally, if time permits, I will show how generic Muchnik reducibility interacts with relative constructibility.

Although forcing is a crucial tool in this context, no knowledge of forcing is required; I'll cover what I need at the beginning of the talk.

This is joint work with Julia Knight and Antonio Montalban.