Logic Seminar

UNIVERSITY OF NOTRE DAME
Department of Mathematics

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Friday, October 31, 2014
1:00 pm
Room: 258 Hurley Hall

Title: Computability strength of $\mathbb{R}$ compared to 2^ω

Abstract:
In recent work, Noah Schweber defines a reducibility notion for structures $\mathcal{A}$ and $\mathcal{B}$, potentially uncountable. The idea of the reducibility is that $\mathcal{A} \leq^*_\omega \mathcal{B}$ if, after a forcing collapse that causes $\mathcal{A}$ and $\mathcal{B}$ to become countable, every copy of $\mathcal{B}$ computes a copy of $\mathcal{A}$. This notion turns out to be independent of the specific forcing that is used.

From this point of view, we wish to consider two structures: the field $\mathcal{R} = (\mathbb{R}, +, \cdot, <)$, and the two-sorted power set of ω, $\mathcal{W} = (\mathcal{P}(\omega) \cup \omega, \in, S)$. After forcing collapse, $\mathcal{R}$ continues to be an Archimedean real closed field, and ω continues to be ω, so $\mathcal{W}$ continues to be a Scott set.

Schweber proved that $\mathcal{W} \leq^*_\omega \mathcal{R}$. (The binary expansions of the elements of $\mathcal{R}$ are precisely the subsets of ω that can be found in $\mathcal{W}$, before and after forcing collapse.) We prove that the converse does not hold: in any forcing extension of V that collapses 2^ω, there is a copy of $\mathcal{W}$ that cannot compute a copy of $\mathcal{R}$.

This proves that the computability theorist's $\mathbb{R}$ is different from the analyst's $\mathbb{R}$ in a fairly intrinsic manner: the various problems that arise with diadic rationals, or with equality of Cauchy sequences are unavoidable when translating between 2^ω and $\mathbb{R}$.