Topology-Analysis-Geometry Seminar
UNIVERSITY OF NOTRE DAME
Department of Mathematics

Speaker: **Andreas Arvanitoyeorgos**
University of Patras, Greece, and Tufts University

Tuesday, October 7, 2014
11:00 am
Room: 258 Hurley Hall

Title: Progress on homogeneous Einstein manifolds

Abstract:
A Riemannian manifold (M, g) is called Einstein if the Ricci tensor satisfies $\mathrm{Ric}(g) = \lambda g$ for some $c \in \mathbb{R}$. For a Riemannian homogeneous space $(M = G/H, g)$, where G is a Lie group and H a closed subgroup of G, the problem is to classify all G-invariant Einstein metrics. In the present talk I will discuss progress on this problem on two important classes of homogeneous spaces, namely generalized flag manifolds and Stiefel manifolds. A generalized flag manifold is a compact homogeneous space $M = G/H = G/C(S)$, where G is a compact semisimple Lie group and $C(S)$ is the centralizer of a torus in G. Equivalently, it is the orbit of the adjoint representation of G. A (real) Stiefel manifold $V_k\mathbb{R}^n$ is the set of orthonormal k-frames in $\mathbb{R}^n$ and is diffeomorphic to the homogeneous space $SO(n)/SO(n-k)$. One main difference between these spaces is that in the first case the isotropy representation decomposes into a sum of irreducible and *non equivalent* subrepresentations, whereas in the second case the isotropy representation contains equivalent summands. In both cases, when the number of isotropy summands increases, various difficulties appear, such as description of Ricci tensor, G-invariant metrics, as well as solving the Einstein equation, which reduces to an algebraic system of equations. In many cases such systems involve parameters and we use Gröbner bases techniques to prove existence of positive solutions.

Based on joint works with I. Chrysikos (Brno), Y. Sakane (Osaka) and M. Statha (Patras)