Regulations for Doctoral Students in Mathematics

A supplement to the regulations in the Graduate School *Bulletin of Information*

Contents

New Students

A new graduate student should plan to arrive at Notre Dame well before registration. There are orientation events hosted by the Graduate School, and there may be departmental orientation events as well. The student will be assigned a first-year adviser. The student should talk with the Director of Graduate Studies (DGS), with the first-year adviser, and with other faculty members, to get acquainted in the department and to decide on appropriate first-year courses.

Course Requirements

In the first year, a student normally registers for four courses per semester. Four basic courses is a common load, but students are permitted and encouraged to "place out" of basic courses if they arrive with an adequate knowledge of the material (the procedure for doing this is described later). A first-year student not taking four basic courses is expected to take topics courses and/or readings courses to bring the course load to four. Occasionally, there is reason for a student to take a different number of courses, but this needs the approval of the adviser and the DGS.

Basic Courses

60210, 60220 Algebra I, II

60350, 60360 Real Analysis I, II

60370, 60380 Complex Analysis I, II

60330 Basic Geometry and Topology

60440 Basic Topology II

60670 Basic Differential Geometry

60510, 60520 Logic I, II

60610 Discrete Mathematics

60620 Optimization

60650 Basic PDE

The first several of these courses are offered every year in the Mathematics Department. The basic courses provide essential background for later work. Basic courses do not have prerequisites, except that for certain two- semester sequences such as Algebra, Algebra I is the prerequisite for Algebra II. Basic courses all have regular homework and comprehensive examinations.

At the end of these regulations is a syllabus for each of the basic courses, together with references covering this material.

Further courses in Mathematics

Other courses serve to introduce an area of mathematics, but they assume some background. Algebraic Geometry (60710) is better taken in the second year, after some basic algebra and/or complex analysis. Probability (60850) has a pre-requisite of Real Analysis I. The syllabi for these and other similar courses are given after those for the basic courses.

The other courses in the department are directed readings courses (56xxx and 86xxx), topics courses, and Intermediate Geometry and Topology (70330). These do not have fixed syllabi---the same course number is used for courses covering very different material in different years.

A first-year student may be able to place out of one or more basic courses. To do so, the student must convince the faculty member teaching the basic course that he or she has mastered the material on the syllabus. A student who places out of a basic course substitutes a readings course or a topics course. Thus, by placing out of one or more basic courses, the student may be able to accelerate the process of beginning research.

A first-year student should select basic courses carefully, with the help of the adviser and other faculty members. A student who is undecided about a research area, or is trying to choose among several areas, must choose courses with special care, to keep open all of the options.

To be full-time in the Graduate School, a student must be registered for at least 9 credits. Above, there is a description of the usual schedule for the first year student, and below there is a description of what happens in the second year and after.

The selection of an area of specialization normally occurs by the end of the first year or the beginning of the second year. Students should explore possible areas of research as soon as possible, by taking topics and readings courses, attending departmental colloquia and seminars, and talking with various faculty members.

In the second year, a student normally registers for three courses per semester. These may be a combination of basic and topics courses. A student who has not yet passed the oral candidacy examination may register for a 3- credit readings course with a possible adviser.

After the second year, a student may be full-time by registering for 9 credits of research and dissertation. However, most students should continue to register for interesting or important courses through the full time of study at Notre Dame.

A student's schedule must always have the approval of the adviser, and anything unusual must also be approved by the DGS.

Course requirements are summarized as follows.

1. A student must, by the end of the first year, complete or place out of 6 basic courses, approved by the adviser, to be counted as the written candidacy examination.

2. A student must accumulate at least 36 credits in basic and topics courses, or possibly, directed readings, during the first three years at Notre Dame. There is no transfer of credits. (Fourth and fifth year students should continue to register for topics courses, as appropriate.) The Graduate School requires at least 60 credits for graduation. This includes credits from the research and dissertation course.

3. Maintain an average G.P.A. of at least B (3.0). This is a condition for being in good academic standing.

4. All students are expected to attend departmental colloquia. Students are also expected to participate in seminars related to their mathematical interests.

Advisers

A new student has an initial adviser assigned by the DGS. As soon as feasible, the student should be working with a permanent adviser/thesis director. Sometimes the initial adviser ends up as the thesis director, but often there is a change in adviser. Well before the oral candidacy examination, the student needs an adviser who has agreed at least to supervise the student's preparation for this examination.

To change advisers, the student needs the consent of the new adviser. In addition, the student should inform the DGS and, as a courtesy, the former adviser.

After passing the oral candidacy examination, a student may wish to change area as well as adviser. There is no problem in doing this so long as the student has taken the appropriate courses and has the consent of the new adviser. The student need not re-take the candidacy examinations.

The student should always feel free to come to the adviser, or the DGS, to talk about various aspects of academic life.

Candidacy Examinations

The candidacy examination has two components: written and oral.

Written Candidacy Examination

In the Mathematics Department, the written work in the basic first-year courses is counted as the written candidacy examination. The typical student passes the written candidacy examination by completing at least 6 basic three-credit courses, each with a grade of *B* or better (not *B*-). A student who places out of the first semester of a basic course sequence and then takes the second semester with a grade of *B* or better, will be allowed to count both semesters toward the written exam. Alternatively, a student who places out of a basic course may earn the right to count this course toward the written candidacy examination by taking a final exam in the course and getting a grade of *B* or better on that exam. The written candidacy requirement must be completed in the first year.

Oral Candidacy Examination

The oral candidacy examination, taken after the written candidacy examinations are completed, focuses on an "advanced" topic. This may be related to a topics course or seminar, or it may come from advanced research texts or articles. In any case, the student should begin working on the advanced topic, with an advisor, well in advance of the examination. The material to be counted as the advanced topic must have the approval of the adviser and the DGS.

The board of examiners for the oral candidacy examination consists of four examiners from the Mathematics Department. The members of the examining board are selected by the DGS (based on suggestions of the student and adviser). Normally, the adviser is one of the examiners.

The topic for the oral candidacy examination should be chosen months before the examination. The syllabus for the oral candidacy examination must be made available to all members of the examining board at the time they agree to serve. All examiners should restrict their questions to the advanced topic or other material on the given syllabus. Thus, the syllabus should provide guidance to the examiners.

The oral candidacy examination begins with a presentation by the student lasting between 30 and 50 minutes (the time should be set by the adviser within this interval). This is followed by questions on material from a syllabus related to the presentation. The examination lasts from one and a half to two hours. After the completion of the examination, the four examiners vote "pass" or "fail." A vote of "pass" means that, in the eyes of the particular examiner, the student has passed all parts of the examination. The student is considered to have passed the oral candidacy examination only if at least three of the four examiners vote "pass". In case the student does not pass, the examiners vote whether to recommend the student for a master's degree. The student is informed of the outcome of the examination immediately.

The oral candidacy examination is usually held in early September, late November, late January, and/or April. Students are encouraged to take the examination as early as possible. In general, students must take the oral candidacy examination by the end of January in the second year. In special circumstances, a short delay may be allowed, with the permission of the DGS. Students must pass the oral candidacy examination no later than April of the second year.

In the Mathematics Department, there is no requirement for a "doctoral dissertation proposal." The material for the advanced topic may include specific research problems and partial results, but in most cases, the candidacy examination comes at a time when the student can only propose to work in a certain area.

The Thesis

Thesis research, under the supervision of the thesis director, normally begins after the successful completion of the candidacy examination**s**. The thesis director is expected to be concerned with the interest and significance of a thesis topic, with the originality of the research, and with the accuracy and the style of the manuscript. The final draft of the thesis should provide enough background and detail to make for easy reading by a semi-expert in the area, but should also be in a form that can easily be edited and shortened for publication, so that it would be suitable for publication in a good mathematics journal.

After the thesis director has approved the thesis, it is submitted to three official readers. Normally, these are professors in the Mathematics Department at Notre Dame (Sometimes, however, experts from other universities serve.). The DGS must approve the choices. After all three official readers have approved the thesis, the thesis defense can be scheduled. In approving the thesis, the official readers certify that it is worthy of defense. They may continue to require changes.

Thesis Defense

The thesis defense is an oral examination on the contents of the thesis and its relation to other work in the same area. The board of examiners for the thesis defense normally consists of four examiners, where these are the thesis director and the three official readers. Sometimes, there are two advisors.

The examination begins with a 30-50 minute presentation by the Ph.D. candidate, prepared in consultation with the thesis director (who also sets the length). A round of questions follows this by the examiners. There may be questions about specific points in the thesis, and also about the importance of the research and what further work it suggests. A thesis defense is public, in the sense that people other than the candidate and the members of the board of examiners may be present for the lecture. Such people leave the room prior to the vote. Voting is as for the oral candidacy examination. As for the oral candidacy examination, the candidate is informed of the outcome immediately.

After a successful defense, the candidate may still need to make minor changes in the thesis. Then the final version of the thesis, signed by the thesis director, is submitted to the Graduate School.

Getting the thesis read and approved, scheduling the thesis defense, making corrections, and having the thesis accepted by the Graduate School is a time-consuming process that requires strict adherence to the timetables set by the Department of Mathematics and the Graduate School. The thesis must be submitted to the readers well before the Graduate School deadline for submission of theses. The latter is roughly two months before the graduation date. August graduation entails special difficulties, since there are fewer faculty members available during the summer to serve as official readers.

There are strict rules about formatting, margins, etc., which must be observed if the thesis is to be accepted by the Graduate School. The Ph.D. candidate should be sure to consult the Graduate School's *Guide for Formatting and Submitting Doctoral Dissertations and Master's Theses* (available on the Graduate School's website at http://graduateschool.nd.edu/resources-for-current-students/dt/). This document changes from year to year, so it is important to consult the current version and other students who have recently written theses.

Summary of Official Requirements for the Ph.D. Degree

1. Courses and credits– appropriate basic courses, 36 credits in basic and topics courses, plus further credits to make a total of 60 credits.

2. Residency– 4 consecutive semesters of full-time study (as required by the Graduate School). The term ``full-time'' means that the student is registered for at least 9 credits, and that the schedule is approved by the adviser.

3. Written and oral candidacy examinations

4. Admission to Degree Candidacy (see the Graduate School *Bulletin of Information*)

5. Thesis carried through the following steps:

   a. approved by the adviser and all three readers
   b. prepared for electronic submission
   c. signed by adviser
   d. accepted by the Graduate School

6. Thesis Defense (this cannot be officially scheduled until steps 1-4 and 5a are completed).

Student Status

The Mathematics Department does not admit students who plan to study on a part-time basis.

A student is considered to be full-time if he or she is registered for at least 9 credits and the adviser certifies that the student is working full-time. At first, the work is almost entirely tied to courses. Later, the thesis is the main focus. In some cases, the student may simply register for 9 credits of research and dissertation.

To be in good academic standing, a student must maintain a G.P.A. of at least 3.0 and be on schedule in terms of course work and examinations. In addition, once the student is no longer registered for basic courses, the adviser must indicate that the student is making satisfactory progress. Thus, it is essential for the student to keep the adviser informed about his or her progress. Normally, a student who is not registered for basic courses is registered for research and dissertation with an adviser, and the adviser indicates that the student is making satisfactory progress by giving a grade of at least B in this course.

A student who does not succeed in passing the first year courses is not permitted to continue in the second year. A student who does not pass the candidacy examinations by the end of the second year is not permitted to continue in the third year. A student who, for two consecutive semesters, fails to maintain good academic standing (as described above) is not permitted to continue further.

A student must fulfill all doctoral requirements, including the dissertation and its defense, within eight years from the time of matriculation. Failure to complete any of the Graduate School or departmental requirements within the prescribed period results in forfeiture of degree eligibility. A student is normally expected to finish within five years.

The M.S. Degree

The graduate program in the Mathematics Department is almost entirely a Ph.D. program. Students are not normally admitted directly to a Master's program. There is a Master of Science in Interdisciplinary Mathematics degree, for students who do not need funding and wish to pursue an interdisciplinary project, or to carry out serious mathematical work while pursuing a Ph.D. in another department. (The requirements for the MSIM are available through the DGS.)

A student who is working toward a Ph.D. in Mathematics may qualify for a Master of Science degree along the way, if he or she has accumulated 30 credit hours, has passed the written candidacy examination, and has either passed the oral candidacy examination or (without passing) exhibited sufficient knowledge to obtain a positive recommendation from the examiners. Having met the requirements, a student must also ask to have his or her name put on the graduation list. The Master of Science degree is not given automatically.

Financial Support

Continued financial support requires the student to maintain good academic standing, and to carry out in a conscientious way any teaching duties associated with the support. The Mathematics Department does not provide financial support beyond the fifth year. Neither teaching assistants nor fellowship holders are allowed to take outside employment without special approval. (Occasionally, students are permitted to tutor up to four hours per week, with the permission of the adviser and the Graduate School.)

Decisions about financial support are made at the end of the spring semester for the next academic year. Students are informed about support at that time.

Teaching Opportunities

First year students have no teaching duties. Second and third year students are typically assigned to conduct tutorial. After gaining some experience with the tutorial sessions, students are assigned to teach a course—usually one section of a multi-section introductory course, under the supervision of a senior faculty member.

The Director of Graduate Studies

The DGS has direct responsibility in the following areas:

1. Providing information about the program to prospective and current graduate students

2. Assigning initial advisers, and overseeing course placement

3. Selecting examiners for oral candidacy examinations (for approval by the Graduate School)

4. Overseeing the scheduling of thesis defenses.

The department’s Associate Chair makes teaching assignments.

The Department Chair and the DGS, in consultation with the adviser, when appropriate, make recommendations for fellowships and stipends.

Syllabi for Basic First-year Courses

The syllabi for the basic courses are given below, with references for each subject.

Algebra I, II

The examinable material for the graduate algebra candidacy exam is 1 through the first part of 3 below (up to but not including categories), though Algebra I will usually cover more than this. Topics labeled *, and perhaps additional topics not mentioned, may be covered at the discretion of the instructor.

1. Groups
Groups. Cyclic groups, permutation groups symmetric and alternating groups, matrix groups. Subgroups, quotient groups, direct products, homomorphism theorems. Automorphisms, conjugacy. Cosets, Lagrange's theorem. Group actions, G-sets, Sylow theorems, free groups and presentations.

2. Rings
Rings. Polynomial rings, matrix rings. Ideals, quotient rings, homomorphism theorems. Prime and maximal ideals. Principal ideal domains, Euclidean domains, unique factorization domains. Localization of rings, field of fractions.

3. Modules
Finite dimensional vector spaces. Basis, dimension, linear transformations, matrices. Modules. Submodules, quotient modules, homomorphism theorems, structure of finitely generated modules over a principal ideal domain. Language of categories and functors. Direct sums and products, free, projective and injective modules. Duality, multilinear forms, determinants.

4. Canonical forms of matrices of linear transformations
Jordan, rational and primary rational canonical forms. Invariant factors and elementary divisors.

5. Fields
Fields, algebraic and transcendental field extensions, degree, transcendence degree, algebraic closure.

Fundamental theorem of Galois theory, separability, normality. Finite fields. *Cyclotomic extensions, *cyclic extensions, *solvable and nilpotent groups, *Impossibility proofs (trisecting angles, etc), *solvability of polynomial equations by radicals

6. Tensor products
Tensor products, *algebras, *(the tensor, symmetric and exterior algebras)

7. Chain conditions
Artinian and Noetherian rings and modules, Hilbert basis theorem. *Simple and semisimple modules.

8. Commutative algebra
Localization of modules. Integral ring extensions. *Localization and spec of a ring.
*Nakayama's lemma, *Noether normalization lemma. *Nullstellensatz. *Affine algebraic varieties and commutative rings.

References

T. Hungerford, *Algebra, Graduate Texts in Mathematics 73.*

N. Jacobson, *Basic algebra I, II*. S. Lang, *Algebra*.

Real Analysis I, II

1. Calculus
Calculus of one and several variables, the Implicit and Inverse Function Theorems, pointwise and uniform convergence of sequences of functions, integration and differentiation of sequences, the Weierstrass Approximation Theorem.

2. Lebesgue measure and integration on the real line
Measurable sets, Lebesgue measure, measurable functions, the Lebesgue integral and its relation to the Riemann integral, convergence theorems, functions of bounded variation, absolute continuity and differentiation of integrals.

3. General measure and integration theory
Measure spaces, measurable functions, integration convergence theorems, signed measures, the Radon-Nikodym Theorem, product measures, Fubini's Theorem, Tonelli's Theorem.

4. Families of functions
Equicontinuous families and the Arzela-Ascoli Theorem, the Stone-Weierstrass Theorem.

5. Banach spaces
L_P- spaces and their conjugates, the Riesz-Fisher Theorem, the Riesz Representation Theorem for bounded linear functionals on L_P, C(X), the Riesz Representation Theorem for C(X), the Hahn-Banach Theorem, the Closed Graph and Open Mapping Theorems, the Principle of Uniform Boundedness, Alaoglu's Theorem, Hilbert spaces, orthogonal systems, Fourier series, Bessel's inequality, Parseval's formula, convolutions, Fourier transform, distributions, Sobolev spaces.

References

Apostol, *Mathematical Analysis.*

Knapp, *Basic Real Analysis.*

Riesz-Nagy, *Functional Analysis.*

Royden, *Real Analysis.*

Rudin, *Principles of Mathematical Analysis.*

Rudin, *Real and Complex Analysis.*

Rudin, *Functional Analysis.*

Simmons, *Introduction to Topology and Modern Analysis.*

Wheeden-Zygmund, *Measure and Integration.*

Folland, *Real Analysis.*

Real Analysis I covers the material on calculus, and Lebesgue measure and integration. It is roughly Chapters I-III and V-VI of Knapp. The remaining material is in Real Analysis II.

Complex Analysis I, II

I. Winding number, integral along curves. Various definitions of a holomorphic function. Connection with harmonic functions. Cauchy Integral Theorems and Cauchy Integral Formula for closed curves in a domain, and for the boundary of a domain, Poisson Formula. The integral of a holomorphic function and its dependence on the path of integration. Open Mapping Theorem, Inverse Function Theorem, maximum and minimum principle, Liouville's Theorem. Uniform convergence of holomorphic functions. Normal families of holomorphic functions. Montel and Vitali Theorems, Picard's Theorem. Power series, Laurent series. Residues and classification of isolated singularities, meromorphic functions. Divisor of a meromorphic function. Residue Theorem, argument principle, Rouché's Theorem, computation of integrals. Riemann Mapping Theorem, argument principle. Möbius maps. Schwartz Lemma. Theorems of Mittag-Leffler and Weierstrass. Gamma Function, Riemann Zeta Function, Weierstrass $\wp$-function.

II. Definition of complex manifolds and examples. Riemann surfaces. The concepts of divisors, line bundles, differential forms and Chern forms. The Riemann-Roch Theorem. The Dirichlet problem for harmonic functions. The concept of genus of a Riemann surface.

References

Ahlfors, *Complex Analysis*.
Burchel, *An Introduction to Classical Complex Analysis I.*

Conway, *Functions of One Complex Variable*.

Forster, *Lectures on Riemann Surfaces.*

Gunning, *Lectures on Riemann Surfaces.*

Knopp, *Theory of Functions I, II, and Problem Books*.

Complex Analysis I covers approximately Chapters 1-6 of Ahlfors. Complex Analysis II covers the remaining material.

Basic Geometry and Topology (Fall Semester)

1. Point-set topology (a quick review): topological spaces, subspaces, quotients and products. Properties of topological spaces: Hausdorff, compact, connected. Examples: spheres projective spaces, homogeneous spaces, CW-complexes.

2. Basic algebraic topology: fundamental group, covering spaces, the van Kampen Theorem, the fundamendal group of a surface.

3. Smooth manifolds: tangent bundle and cotangent bundle, vector bundles, constructions with vector bundles (sums, products symmetric and exterior powers), sections of vector bundles, differential forms, the de Rham complex, inverse and implicit function theorems, transversality.

References

Munkres, *Topology*.

Hatcher, *Algebraic Topology (Chapter 1)*.

Lee, *Introduction to Smooth Manifolds.*

Basic Topology II (Spring Semester)

1. Homology: singular homology, the Eilenberg-Steenrod axioms, homology group of spheres, the degree of a map between spheres, homology calculations via CW complexes, proof of homotopy invariance, proof of excision, universal coefficient and Kunneth Theorem.

2. Cohomology: the cup product, the cohomology ring of projective spaces.

3. Poincare duality.

Reference

Hatcher, *Algebraic Topology*

Basic Differential Geometry (Spring Semester)

1. Connections in vector bundles: Covariant derivative, parallel transport, orientability, curvature, baby Chern-Weil.

2. Riemannian geometry: Levi Cevita connection, exponential map, Jacobi fields, arc length variation formulas, fundamental equations for metric immersions and submersions, space forms, Hopf-Rinow, Hadamard-Cartan, Bonnet-Myers (Gauss-Bonnet, Bochner technique).

3. Other geometric structures: (one or more of Kahler manifolds, symplectic manifolds, contact manifolds).

References

Chavel, *Riemannian Geometry: A modern introduction.*

Grove, *Riemannian geometry: A metric entrance.*

Petersen, *Riemannian Geometry.*

Gallot, Hulin, and Lafontaine, *Riemannian geometry.*

Cullen, *Introduction to General Topology.*

Dugundji, *Topology.*

Kelley, *General Topology.* Munkres, *Topology.*

Steen, *Counterexamples in Topology.*

Logic I, II

1. Model Theory
First order predicate logic. Structures and theories. Compactness theorem. Ultraproducts. Löwenheim-Skolem theorems. Saturation and homogeneity. Quantifier elimination: methods, examples, and applications. Types and type spaces. Prime and Saturated models. Countable models and countable categoricity. Henkin constructions should be seen, for example in the omitting types theorem.
If time permits, more advanced topics may be included.

References

C. C. Chang and H. J. Keisler, *Model Theory*.

D. Marker, *Model theory, an introduction*.

K. Tent and M. Ziegler, *A course in model theory*.

2. Computability Theory
Turing machines. Primitive recursive functions, partial recursive functions, equivalence of Turing and Kleene definitions. Recursive sets, computably enumerable sets. The Recursion Theorem. Index sets and Rice's Theorem. Strong reducibilities (*m*-reducibility, 1-reducibility). Relative computability, Turing degrees, jumps, the Kleene-Post Theorem. The arithmetical hierarchy. Computably enumerable degrees, the Friedberg-Muchnik Theorem, the Low Basis Theorem.
If time permits, further topics may be included.

References

R. I. Soare, *Recursively Enumerable Sets and Degrees.*

H. Rogers, *Theory of Recursive Functions and Effective Computability.*

S. B. Cooper, *Computability Theory*

3. Set Theory
Axioms of *ZFC*, Schröder-Bernstein Theorem, ordinals, ordinal arithmetic, proof and definition by recursion. The Von Neumann hierarchy. Cardinals, cardinal arithmetic. Special kinds of cardinals—regular and singular, successor and limit, inaccessible, equivalent versions of the Axiom of Choice, the constructible hierarchy.
If time permits, further topics may be included.

References

T. Jech, *Set Theory*.

K. Kunen, *Set Theory*.

P. Cohen, *Set Theory and the Continuum Hypothesis.*

Logic I covers the material on model theory and part of the material on set theory, including ordinal and cardinal arithmetic, and definitions by recursion. Logic II covers the material on computability, plus more set theory.

Discrete Mathematics

This course provides an introduction to the questions of existence, structure and enumeration of discrete mathematical objects. Topics include:

1. Enumeration — basic counting principles (including permutations, combinations, compositions, pigeon-hole principle and inclusion-exclusion), basic counting sequences (such as binomial coefficients, Catalan numbers and Stirling numbers), and recurrence relations and generating functions.

2. Structure and existence — Graphs (including trees, connectivity, Euler trails and Hamilton cycles, matching and coloring), partially ordered sets and lattices, basic Ramsey theory, error detecting and correcting codes, combinatorial designs, and techniques from probability and linear algebra.

Other topics chosen by the instructor may be included if time permits.

The course will be at the level of the following books:

Stasys Jukna, Extremal combinatorics (with applications to computer science)
Peter Cameron, Combinatorics (topics, techniques and algorithms)
J.H. van Lint and R.M. Wilson, A course in Combinatorics

Optimization

Convex sets. Caratheodory and Radon's theorems. Helly's Theorem. Facial stricture of convex sets. Extreme points. Krein-Milman Theorem. Separation Theorem. Optimality conditions for convex programming problems. Introduction to subdifferential calculus. Chebyshev approximations.

References

Barvinok, *A Course in Convexity*.

R. Webster, *Convexity*.

Basic PDE

Laplace equations: Green's identity, fundamental solutions, maximum principles, Green's functions, Perron's methods. Parabolic equations: Heat equations fundamental solutions, maximum principles, finite difference and convergence, Stefan Problems. First order equations: Characteristic methods, Cauchy problems, vanishing of viscosity-viscosity solutions. Real analytic solutions: Cauchy- Kowalevski theorem, Holmgren theorem.

Reference

L. Evans, *Partial Differential Equations*.

Further Introductory Courses

Introduction to Algebraic Geometry (two-semester course sequence).

This is an introduction to algebraic varieties, schemes, coherent sheaves, curves, and surfaces.

Lie groups and Lie algebras

This course is an introduction to Lie theory and covers material related to Lie groups, Lie algebras, and their applications. Specific topics will be drawn from Lie groups, the Lie algebra of a Lie group, the exponential map, subgroups, the representation theory of Lie groups and Lie algebras, the structure of Lie algebras, root systems, the Weyl group, enveloping algebras, and infinite dimensional Lie algebras.

Probability
(one semester, graduate level, intermediate or basic, prerequisite: first sem. of Real Analysis)

1. Preliminaries:
Kolmogorov's axioms, examples of probability spaces, independence, random variables, probability distributions, expectation, variance, Kolmogorov's consistency theorem

2. Sums of Independent Random Variables:
basic inequalities and modes of convergence (almost surely, in probability, L^p), Rademacher functions, more inequalities (Chernoff, Bernstein, Hoeffding, Bennett, Khinchine, Marcinkiewicz-Zygmund)

3. Laws of Large Numbers:
weak Laws of Large Numbers, uniform integrability, Kolmogorov's inequality, strong Laws of Large Numbers, Levy's Refection principle, Law of the Iterated Logarithm

4. Central Limit Theorem:
Lindeberg-Levy condition, Fourier transform and characteristic functions, weak* convergence of probability measures

5. Discrete Time Martingales:
conditional expectation, submartingales and supermartingales, Doob's inequality, stopping times, Hunt's theorem, martingale convergence theorem

(Optional - time permitting):
6. Large Deviations
7. Markov processes
7. Continuous Martingales
8. Wiener Measure, applications to PDE
9. Marcinkiewicz-Zygmund and Burkholder inequalities, applications to harmonic analysis

10. Elements of Ergodic Theory
11. Metrics on probability measures, applications to statistics
12. Kantorovich-Wasserstein distance and Optimal Transport

References

D. Stroock, *Probability Theory: an analytic view*

A. Borovkov, *Probability Theory*

S. Varadhan, *Probability Theory/Stochatic Processes*

W. Feller, *An Introduction to Probability Theory and its Applications* 2 vols.

Linear Control

Introduction to linear system theory. Linear-quadratic control, H-infinity control, introduction to robust control based on matrix cube theorem, linear matrix inequalities and interior-point algorithms.

References

A. Ben-Tal and A. Nemirovski, *Lectures on Modern Convex Optimization*.

P. Lancaster and L. Rodman, *Algebraic Riccati Equations*.

S. Boyd et. al. *Linear Matrix Inequalities in System and Control Theory*.

Grievance Procedure

If a mathematics graduate student has a grievance, the procedure is to go first to the DGS, unless the grievance involves the DGS, in which case, the student should approach the Department Chair. The DGS, or the Department Chair, will try to work with the parties involved to reach a solution. If this is not sufficient, the Department Chair will appoint an *ad hoc* committee of faculty members and students (not directly involved) to hear a particular case. If the student feels that this is not enough, then he or she may appeal to the Graduate School. The Graduate School *Policies* and *Grievance and Appeal Procedures* can be found in the Graduate School *Bulletin of Information* or on their website at http://graduateschool.nd.edu/resources-for-current-students/.

Departmental Appeal of Dismissal

A graduate student who fails to meet the requirements involving coursework and exams may be automatically dismissed from the program. These requirements are:

1. earning the right to count 6 basic courses toward the written candidacy exams in the first year,

2. completing the oral exam by the end of the second year,

3. earning 36 credit hours in basic and topics (or readings) courses within the first three years, with a GPA of at least 3.0.

There are further grounds for dismissal, which are more subjective---failure to make adequate progress in research, failure to work with the adviser toward the thesis document, failure to perform TA duties properly. Before a student is dismissed for these more subjective grounds, he/she will receive a warning letter from the DGS, with specific steps that need to be carried out by a specified time. At the end of this time, the Graduate Studies Committee will determine whether the steps have been carried out. If the decision is negative, then the student is dismissed. There are very limited grounds for appealing dismissal. The student must show that the decision resulted not from his/her failure to meet expectations, but from personal bias or improper procedures. Since the decision to dismiss involves the DGS, an appeal, if any, starts with the Chair, who will appoint an ad hoc committee, if appropriate. A further appeal to the Graduate School is possible, under very limited conditions. See the Graduate School *Bulletin of Information*.

Graduate Bulletin

Mathematics students are included in all of the general policies and rules of the Graduate School, as given in The Graduate *Bulletin of Information* (http://graduateschool.nd.edu/resources-for-current-students/).