Discrete Mathematics Seminar

UNIVERSITY OF NOTRE DAME
Department of Mathematics

Speaker: **Adam Hammett**
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Monday, November 11, 2013
4:00 pm
Room: 258 Hurley Hall

Title: Trivial meet and join within the lattice of Alternating Sign Matrices

Abstract:
It has been more than three decades since the tantalizing prize of enumerating Alternating Sign Matrices (ASMs) entered the combinatorial landscape, and nearly two decades since the formula was finally proved in 1996, first by Doron Zeilberger then by Greg Kuperberg. Interest in these fascinating objects has only increased since then, with the emergence of new combinatorial objects that ASMs can be transformed into bijectively.

One such object is the Monotone Triangle (MT; Gog Triangle, in Zeilberger's terminology): a left-justified triangular array of positive integers, with i integers in row i and last row $n = (1, 2, ..., n)$, with the property that the integers weakly increase up columns and downward-right diagonals, and strictly increase across rows. Endowed with entry-wise comparisons, the collection of MTs with n rows becomes a partially-ordered set that is a lattice. We deal with the following question: for r independent and uniformly random MTs of order n, how often will their infimum (resp. supremum) be trivial? Here "trivial" refers to the smallest possible MT in the case of infimums, $[(1), (1, 2), (1, 2, 3), ..., (1, 2, 3, ..., n)]$, and the largest possible MT in the case of supremums, $[(n), (n-1, n), (n-2, n-1, n), ..., (1, 2, 3, ..., n)]$. We give sharp asymptotics for the probabilities of these events, showing that in each case the odds are asymptotically $r/A(n)$, for n tending to infinity,! where $A(n)$ is the number of ASMs of order n (equivalently, the number of MTs of order n).