Colloquium

UNIVERSITY OF NOTRE DAME
Department of Mathematics

Speaker: Philippe Di Francesco
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Friday, November 15, 2013
4:00 PM
117 Hayes-Healy Hall

Title: Physics and combinatorics of the octahedron equation: from cluster algebras to arctic curves

Abstract:

The octahedron equation is a non-linear system of recursion relations in discrete 2+1 dimensions, that first appeared in the context of generalized Heisenberg quantum spin chains (T-system). Some solutions of this equation have a representation-theoretic interpretation as q-characters of affine algebras. Other solutions have a purely combinatorial interpretation as generalized Coxeter-Conway Frieze patterns. A key feature of this equation is its discrete integrability, namely the existence of conserved quantities.

This same equation plays a central role in the seemingly unrelated statistical model of domino tilings of a plane square-shaped domain called the Aztec diamond. The tiling configurations display some drastic change of typical behavior between the corners of the domain, where it freezes in certain tiling patterns, and a central region in the bulk, with maximal disorder. In the continuum limit of large size and small mesh, the separation of phases is along the so-called "arctic circle" inscribed inside the square domain.

In this talk, we shall show that the octahedron equation is part of a combinatorial structure called cluster algebra, and how its exact solution may be rephrased in terms of non-intersecting lattice paths, and eventually domino tilings. Using the octahedron equation explicitly, we will proceed and determine various arctic curves depending on initial data. As predicted by Kenyon and Okounkov, we find in general new "liquid" phases within some connected components of these curves.