PDE, Complex Analysis and Differential Geometry

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Title: Quarter-plane problem: the fifth order regularized long wave equation

Abstract:
A class of fifth-order, regularized long wave equations of the form

$$u_t + u_x - \frac{1}{6}\beta u_{xxt} + \delta\beta^2 u_{xxxxt} + \frac{3}{4}\alpha(u^2)_x + \alpha\beta\Big(\gamma_1(u^2)_{xx} + \gamma_2 u_x^2 - \frac{1}{4}u^3\Big)_x = 0 \tag{1}$$

has recently been derived by Bona, Carvajal, Panthee and Scialom. These equations are a higher-order model for small-amplitude, long-crested surface water waves propagating mainly in one direction. Here, the independent variable x characterizes position in the medium of propagation whilst t is proportional to elapsed time. The dependent variable $u = u(x,t)$ is a real-valued function of $x \in \mathbb{R}$ and $t \geq 0$. It represents the deviation of the free surface relative to its undisturbed state at the point x at time t. The subscripts connote partial derivatives while $\delta, \alpha, \beta > 0, \gamma_1, \gamma_2 \in \mathbb{R}$ are modeling constants.

Bona and *el.* have investigated the pure initial-value problem for (1) and showed that it is locally well-posed in $H^s(\mathbb{R})$ for $s \geq 1$. Moreover, if $\gamma_1 + \gamma_2 = 0$, then it is globally well-posed in $H^s(\mathbb{R})$ for $s \geq \frac{3}{2}$.

Assuming the wave is generated in a long water tank with a wave-maker mounted at one end of the tank, the modeling problem becomes an initial-boundary-value problem, namely the so-called the quarter-plane problem which has as natural auxiliary data

$$\begin{cases} u(x,0) = u_0(x), \\ u(0,t) = g(t), \quad u_{xx}(0,t) = h(t) \end{cases} \tag{2}$$

set in $x, t \geq 0$. There are natural consistency conditions, namely

$$u_0(0) = g(0), \quad u_0''(0) = h(0). \tag{3}$$

In this talk, I will address the well-posedness issue for (1) posed as in (2) with the concomitant compatibility condtions (3).