Logic Seminar

UNIVERSITY OF NOTRE DAME
Department of Mathematics

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Thursday, October 3, 2013
2:00 pm
Room: 125 Hayes-Healy Hall

Title: The Generic Degrees of Density-1 Reals

Abstract:
A real is *generically computable* if there is an algorithm which can correctly compute most of the bits of the real (in the sense of asymptotic density), without ever making any mistakes, although potentially without halting everywhere. The original motivation for this concept is derived from recent work in complexity theory which has sought to rigorously address and analyze the phenomenon in which a problem might be much easier to solve in the generic case than it is in the worst case.

We study generic computability from a purely recursion theoretic point of view, and for this reason, we wish to have a degree structure, so that we may study both the difficulty in generically computing a real, and also what the uses are of a generic computation of something that is not generically computable. When we define generic reduction, a transitive notion of generic computability, in the most natural manner, we get a relationship that is $\mathbf{\Pi}^1_1$-complete, and prohibitively difficult to study in general.

The generic degrees of (asymptotic) density-1 reals have a number of interesting properties that make them easier to work with. Also, the Turing degrees embed naturally into the generic degrees, and this gives us another perspective from which we may seek to understand the generic degrees. We present some results explaining why an understanding of the density-1 reals would provide a deeper understanding of the degree structure of the generic degrees in general, and we also show that the hyperarithmetical Turing degrees can be characterized precisely as the Turing degrees which embed below density-1 degrees in the generic degrees.