Title: Stirling numbers and generalizations

Abstract:
The Stirling number of the second kind, \( \binom{n}{k} \), counts the number of partitions of a set of size \( n \) into \( k \) non-empty classes. It also shows up in a whole host of identities, apparently unrelated to combinatorics. For example,

\[
\left(x \frac{d}{dx}\right)^n e^x = \sum_{k=1}^{n} \binom{n}{k} x^k e^x.
\]

Generalizations of the above identity, with \( (x \frac{d}{dx})^n \) replaced by an arbitrary word in the alphabet \( \{x, d/dx\} \), have been considered since at least the 1820’s.

I’ll discuss some of these generalizations, and the associated generalized Stirling numbers, paying particular attention to combinatorial interpretations. To finish I’ll show how a suitable combination of these interpretations leads to a quick resolution of a conjecture that Do Trong Thanh and I made two years ago (but only appeared this week!), concerning asymptotic normality of a certain family of sequences.