

DEFENSE OF THE DOCTORAL DISSERTATION

DEPARTMENT OF MATHEMATICS

“Symbolic Squares of Edge Ideals of d -Partite Hypergraphs”



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Time: 1:00 PM

Location: 209 DeBartolo Hall

Examination Committee:

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Abstract:

This dissertation delves into the realm of squarefree monomial ideals within commutative algebra, spotlighting their significance due to two main factors. First, the process of polarization allows any monomial ideal to be converted into a squarefree monomial ideal, thus preserving crucial algebraic properties and simplifying numerous problems to the classification of squarefree ideals. Second, squarefree ideals exhibit a compelling combinatorial structure closely linked to their algebraic characteristics, providing a robust toolkit for their study. A unique focus is placed on edge ideals of d -partite hypergraphs, a particular class of squarefree monomial ideals generated in a single degree d . The core problem addressed herein is identifying conditions under which the symbolic and regular squares of a d -partite hypergraph's edge ideal coincide. This investigation is pursued through a combinatorial perspective, classifying ideals based on the presence of specific subhypergraphs. Research findings reveal that the alignment of symbolic and regular squares of edge ideals is intrinsically linked to the containment of particular subhypergraph configurations. This insight extends the understanding of edge ideal behavior, providing a novel perspective on their structural dynamics. Additionally, this work includes a comprehensive examination of existing criteria for determining when an edge ideal of a d -partite hypergraph demonstrates a linear resolution. This analysis has been distilled into a combinatorial characterization, reliant on the detection of paths within the corresponding hypergraph.