

Speaker: Dan Burghilea
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Thursday, April 14, 2011
2:00 pm
125 Hayes-Healy Hall

Title: Bar codes and Jordan cells, new topological invariants for circle valued maps

Abstract:

Inspired by the idea of persistent homology we introduce a new class of computable topological invariants for (tame) circle valued maps $f : M \rightarrow S^1$. They are Bar Codes and Jordan Cells. If M is compact smooth manifold and f is topologically tame these invariants are computable by algorithms of relatively low complexity. All familiar topological invariants of interest in Novikov-Morse theory can be recovered from them ; for example the Betti numbers of the fibers, the Betti number of the total space, the Novikov Betti numbers of (M, ξ) , $\xi \in H^1(M; \mathbb{Z})$ representing the homotopy class of f . The Jordan cells permit to define a "Lefschetz zeta function" which among other is responsible for counting the closed trajectories of the gradient of a Morse circle valued map when the gradient is taken with respect to a generic metric.

The definition of these invariants is based on representation theory of quivers (=oriented graphs) initiated by Gabriel, Bernstein-Gelfand Ponomarev, Kac and others . The above theory extends Novikov-Morse theory from Morse circle valued maps to tame maps $f : X \rightarrow S^1$ and even further to 1 co-cycle (the topological version of closed one form on smooth manifolds). This last extension is more elaborate and will not be discussed in this lecture. The theory is motivated by Data Analysis.