COLLOQUIUM

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Will give a lecture entitled

C-compressed sensing and F-coding theory and a bridge when F is a finite field

On

Tuesday, March 29, 2011

at 4:00 PM in Room 129 Hayes-Healy Hall

Abstract:

Compressed sensing or compressive sampling (recovering large data based on a smaller number of measurements) can be cast as a natural error correcting problem with real or complex valued input/output in which an input m-dimensional vector f needs to be recovered exactly, if possible, from corrupted n-dimensional measurements y=Af+e. Here, A is an n by m (coding) matrix and e is an arbitrary and unknown vector of errors. It was shown that under suitable conditions on the coding matrix A, the input f is the unique solution to an l1-norm-minimization problem, called basis pursuit, provided that the support of the vector of errors is not too large. In short, f can be recovered exactly by solving a simple convex optimization problem (which one can recast as a linear program).

The setting for error control coding over a finite field F looks very similar to the above setting of compressed sensing linear programming decoding. But the former requires a finite text and the later the set of reals as the alphabet. Is there any other connection besides this similarity? In our work, we give such a connection which can be used to translate performance guarantees from channel coding linear programming decoding to compressed sensing linear programming decoding. The connection is given by a lemma that maps vectors in the nullspace of some zero-one measurement matrix into vectors of the fundamental cone defined by that matrix. This lemma can be extended from zero-one measurement matrices to complex measurement matrices where the absolute value of every entry is a non-negative integer. This talk will give an introduction to compressed sensing and coding theory (mostly classical linear algebra) and show how they can be described through the same formal setting; we build to the formalized connection given by our bridging maps.

Basis pursuit was introduced empirically in the sciences (e.g., in seismology by Claerbout-Muir and others) in the 1970s, and then studied mathematically in the 1990s by by Tao, Chen, Donoho, Huo, Logan, Saunders, and others. Near-optimal performance guarantees emerged in the 2000s by Cand{\'e}s-Romberg-Tao, Donoho, and others. The field exploded in the last 6 years with the results by Cand{\'e}s-Tao 2005. It has applications in image processing, MRI, sensor networking, geology, astronomy, etc. It is advantageous whenever signals are sparse in a known basis, measurements or computations at the receiver end are expensive, but computations at the receiver end are cheap.