Title: Crystal graphs and SB-labelings

Abstract:

Crystal graphs are an important tool to study the representation theory of Kac-Moody algebras. The crystal graphs arising from highest weight representations give rise to partially ordered sets. In joint work with Cristian Lenart, we study these crystal posets. We prove two types of positive results for those poset intervals which include the highest weight vector of the representation: (1) a crystal operator analogue of the statement that any two reduced expressions for the same Coxeter group element are connected by braid moves, and (2) poset topology and Möbius function results analogous to those of weak Bruhat order. These results rely upon a new, elementary description of the key map. We also provide examples demonstrating that both types of results fail arbitrarily badly for arbitrary intervals in crystal posets coming from highest weight representations, even in type A. The first such examples were found by computer, specifically by Möbius function computations. The first example with unexpectedly large Möbius function was also the first example where (1) failed. Recently this link between (1) and (2) has been explained using the theory of SB-labelings, a theory we developed in separate joint work with Karola Meszaros. The talk will tell this story, providing background along the way.