Title: Infinite Loop Spaces in Algebraic Geometry

Abstract:

A classical theorem in modern homotopy theory states that functors from finite pointed sets to spaces satisfying certain conditions model infinite loop spaces (Segal 1974). This theorem offers a recognition principle for infinite loop spaces. An analogous theorem for Morel-Voevodsky's motivic homotopy theory has been sought for since its inception. In joint work with Marc Hoyois, Adeel Khan, Vladimir Sosnilo and Maria Yakerson, we provide such a theorem. The category of finite pointed sets is replaced by a category where the objects are smooth schemes and the maps are spans whose "left legs" are finite syntomic maps equipped with a K-theoretic trivialization of its cotangent complex. I will explain what this means, how it is not so different from finite pointed sets and why it was a natural guess. In particular, I will explain some of the requisite algebraic geometry. Time permitting, I will also provide 1) an explicit model for the motivic sphere spectrum as a torsor over a Hilbert scheme and, 2) a model for all motivic Eilenberg-Maclane spaces as simplicial ind-smooth schemes. I will arrive on Monday and leave on Wednesday morning.