Abstract:

In the 90s Crane--Frenkel proposed the construction of a 4d TQFT from a bimonoidal category, that is, a category equipped with both a monoidal and comonoidal structure so that the data defining the comonoidal structure is itself monoidal. Despite being an influential proposal it took nearly 20 years for the first interesting examples of bimonoidal categories to appear. In this talk I will describe how to extend the classical construction of the Hall algebra of an abelian category to produce new examples of bimonoidal categories: the Hall bimonoidal categories. The approach we take is based on a new perspective on the Hall construction introduced by Dyckerhoff--Kapranov and Gálvez-Carrillo--Kock--Tonks. In this talk I will give a direct construction of the Hall bimonoidal categories. I begin by describing how the Hall construction can be broken into three steps: First taking the Waldhausen S-construction of an abelian category $A$ to produce a 2-Segal space $S(A)$, then building the universal Hall algebra of $S(A)$ which, in the final step, is linearised. Our construction of the Hall bimonoidal category proceeds by modifying each of these three steps: Iterating twice the Waldhausen S-construction to get a so-called double 2-Segal space, building from the double 2-Segal space a universal Hall bialgebra, and then applying a categorical linearisation.