Title: Equilateral and almost-equilateral sets in $n$

Abstract:
Combinatorics may be defined narrowly as “the art of counting”, more broadly as the study of discrete (finite or countable) structures, and even more broadly as “[the class of] problems that it is reasonable to attack more or less from first principles” (this last definition is due to Tim Gowers). However it’s defined, it’s a subject filled with easy to state problems whose solutions require drawing on many areas of non-combinatorial mathematics — probability, linear algebra, differential equations, . . . . I’ll illustrate this by discussing some questions in discrete geometry, including the well-known “how large can a set of points in $n$ be, if the distance between any pair of points is 1?”, and the much less well-known “what if the distance between any pair is merely required to be close to 1?” (The answer to this latter may be surprising.)