Title: Understanding the Paris-Harrington Theorem, Part II

Abstract:
Call a finite set of natural numbers \( X \) relatively large if \( |X| \geq \text{min}(X) \). Given natural numbers \( M, e, r, \) and \( k \), let \( M \rightarrow (k)^e_r \) denote the statement that for every partition \( P : [M]^e \rightarrow r \) of the size \( e \) subsets of \( M \) into \( r \) colors, there is a relatively large set \( X \subseteq M \) that is homogeneous for \( P \). It is not difficult to show, using the infinite Ramsey theorem and a compactness argument, that

\[
(\ast) \quad (\forall e, r, k)(\exists M)(M \rightarrow (k)^e_r).
\]

However, Paris and Harrington\(^1\) showed that the statement \((\ast)\) cannot be proved in Peano arithmetic. We will try to present a clear explanation of their proof.

\(^1\)A Mathematical Incompleteness in Peano Arithmetic, Handbook of Mathematical Logic, 1977