

TOPICS IN ALGEBRA, 80210, FALL 2015, TuTh 12:30–1:45; time can be negotiated

Representation theory of reductive groups, Sam Evens

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Let G be a group. A representation of G is a group homomorphism $\phi : G \rightarrow GL(V)$ from G to the general linear group $GL(V)$ over some vector space V , which is possibly infinite dimensional.

Topics to cover:

- (1) Representations of the symmetric group
- (2) Representations of $GL(2, F)$, where F is a finite field.
- (3) Representations of compact Lie groups
- (4) Representations of $GL(2, \mathbf{R})$, plus survey on real reductive groups
- (5) Representations of $GL(2, F)$, where F is a p-adic number field, plus survey on reductive p-adic groups.

I will give some suggested exercises, but these will be optional.

PREREQUISITES: In principle, this course should be understandable to a student who has completed the first year of graduate algebra (60210-60220), but the class will be easier to understand for students who have had some familiarity with Lie groups. For representations of continuous groups, some familiarity with the language of Hilbert spaces is useful, but not really necessary.

SOME REFERENCES: Daniel Bump, Automorphic forms and representations

William Fulton, Young Tableaux

Joe Harris and William Fulton, Representation theory, a first course

Anthony Knapp, Representation Theory of Semisimple Groups: An Overview Based on Examples (PMS-36).

Pierre Cartier, La conjecture locale de Langlands pour $GL(2)$ et la démonstration de Ph. Kutzko. (French) [The local Langlands conjecture for $GL(2)$ and Ph. Kutzko's proof] Bourbaki Seminar, Vol. 1979/80, pp. 112138, Lecture Notes in Math., 842,

S. Kudla, The local Langlands correspondence: the non-Archimedean case. Motives (Seattle, WA, 1991), 365-391, Proc. Sympos. Pure Math., 55, Part 2, Amer. Math. Soc., Providence, RI, 1994

I'll also use a number of online lecture notes.