PDE, Complex Analysis and Differential Geometry Seminar

UNIVERSITY OF NOTRE DAME
Department of Mathematics

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Tuesday, April 14, 2015
2:00 PM
258 Hurley Hall

Title: On Bispectrality in a Non-commutative Context

Abstract:

Operators on functions in two distinct sets of variables are said to be "bispectral" when they share an eigenfunction depending on both sets so that the eigenvalues for the operator in each set depend (non-trivially) on the other variables. Bispectrality has been linked to integrable
systems in three ways: bispectral operators are found to be special Lax operators for integrable hierarchies (e.g. rational KP solutions), the motion of poles of the two operators under the integrable dynamics are governed by an integrable classical particle system and its dual respectively, and the quantized versions of those dual Hamiltonians are themselves bispectral sharing an eigenfunction.

Most of the previous results on bispectrality apply to differential and difference operators with scalar coefficients. This talk will concern ongoing efforts to study bispectrality of differential and difference operators whose coefficients are themselves from a non-commutative ring (for example, matrix differential operators). The results to be presented include a theorem describing a large class of bispectral solutions to the ncKP hierarchy. These are different from previously presented bispectral solutions to the matrix KP hierarchy in that the eigenvalues themselves are non-commuting (whereas previously the eigenvalues were always either scalar or at least diagonal). Moreover, informative counter-examples will be presented demonstrating that some things we have come to expect from the scalar case are no longer true for bispectrality in the non-commutative context. In particular, we will see why it is no longer true that all rational Darboux transformations preserve bispectrality, why the "ad-nilpotency" which has been a useful tool in studying bispectrality since the very first paper does not always work, and why the non-commutative approach reveals that the "anti-isomorphism" used in the commutative context may actually have been an isomorphism all along.