Abstract:

Many mathematical principles can be stated in the form "for all X such that C(X) holds, there is a Y such that D(X,Y) holds", where X and Y range over second order objects, and C and D are arithmetic conditions. We think of such a principle as a problem, where an instance of the problem is an X such that C(X) holds, and a solution to this instance is a Y such that D(X,Y) holds. Examples of particular relevance to this talk are versions of Koenig's Lemma (such as KL and WKL) and of Ramsey's Theorem (such as RT^n_2). We'll discuss several notions of computability theoretic reducibility between such problems, and their connections with reverse mathematics. Among other things, I will explain how recasting the idea of "every omega-model of P is a model of Q" in terms of games allows us to define a notion of uniform reducibility from Q to P that permits the use of multiple instances of P to solve a single instance of Q. This is joint work with Carl Jockusch.