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Topics in Logic: The hyperarithmetical hierarchy and computable structure theory

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Davis and Mostowski independently defined an extension of the arithmetical hierarchy, the *hyperarithmetical hierarchy*, with levels running through the computable ordinals. Ash isolated a special collection of $L_{\omega_1\omega}$-formulas, the *computable infinitary formulas*, in which the infinite disjunctions and conjunctions are over computably enumerable sets. These formulas have proved to be useful in computable structure theory. I will present some results, and I will mention some open questions.

In computable structure theory, we may ask how hard it is to describe a class of countable structures, closed under isomorphism. By a result of Vaught, a class is $\mathbf{\Sigma}_\alpha$ in the *Borel hierarchy* if and only if it is axiomatized by a Σ_α sentence of $L_{\omega_1\omega}$. Vanden Boom showed that a class is Σ_α in the *effective Borel hierarchy* if and only if it is axiomatized by a computable Σ_α sentence. We consider how hard it is to describe a single structure, up to isomorphism. There are computable structures with computable infinitary Scott sentences of various complexities. There are also computable structures, such as the Harrison ordering, with no computable infinitary Scott sentence.

In computable structure theory, we are also interested in the information coded in a structure. We ask what is needed to build an isomorphic copy of the structure. Wehner and Slaman (independently) gave examples of structures with copies computable in every non-computable set, but no computable copy. More recently, Greenberg, Montalbán, and Slaman gave an example of a structure with copies computable in every hon-hyperarithmetical set, but no hyperarithmetical copy.