MATH 80750: TOPICS IN DIFFERENTIAL GEOMETRY - SPRING 2015
SEMI-CLASSICAL ANALYSIS & APPLICATIONS

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MWF 12:50-1:40

Semi-classical analysis is the study of the correspondence between classical and quantum mechanics in a precise mathematical framework. The main part of the study involves the construction of oscillatory solutions (WKB) to the Schroedinger equation from the underlying Hamiltonian dynamics/classical mechanics. These solutions are roughly coming from an application of the stationary phase formula to the Feynman path integral. Although used in mathematical physics such a study has a variety of applications within pure mathematics. Examples include

(1) Spectral Geometry: What geometric/topological properties of a Riemannian manifold (e.g. volume, total curvature, lengths of geodesics) can be described in terms of the eigenvalues of its Laplacian? Hormander's sharp Weyl Law.
(2) Heat kernel asymptotics related to (Atiyah-Singer) index theorems.
(3) Witten's proof of Morse inequalities via supersymmetry.
(4) Complex Geometry: Kodaira embedding, Bergman kernel expansion, Kahler quantization.
(5) Quantum ergodicity theorems describing limits of eigenfunctions for chaotic/Anosov flows.
(6) Complex analogue of Hilbert's seventeenth problem.

We shall need to go through the basics of Symplectic geometry (Darboux theorems, Hamiltonian vector fields, Lagrangian submanifolds, the symplectic category, the Maslov index) in order to give a mathematical description of classical mechanics. The mathematics behind the quantum mechanics is mostly described in the theory of pseudodifferential operators and their generalizations (Fourier integral operators). I expect to spend most of the time giving these mathematical descriptions and then going through as many as the above applications as the time permits/audience wishes.

There is no single text for the entire course although the books of Guillemin-Sternberg[2], Dimassi-Sjostrand[1] and Zworski[3] on Semi-classical analysis are good references. It shall be useful to have some prior knowledge of differentiable manifolds and functional analysis. The course should be of interest to differential geometry/topology, mathematical physics and analysis/PDE students.

References

[1] Mouez Dimassi and Johannes Sjöstrand. *Spectral asymptotics in the semi-classical limit*, volume 268 of *London Mathematical Society Lecture Note Series*. Cambridge University Press, Cambridge, 1999.
[2] Victor Guillemin and Shlomo Sternberg. *Semi-Classical Analysis*. International Press of Boston, 2013.
[3] Maciej Zworski. *Semiclassical analysis*, volume 138 of *Graduate Studies in Mathematics*. American Mathematical Society, Providence, RI, 2012.