The analogy between numbers and functions is one of the guiding principles of number theory today. Recently this analogy was enriched by the consideration of an arithmetic analogue of differential calculus, in which derivative operators acting on functions are replaced by Fermat quotient operators acting on numbers. An arithmetic analogue of the theory of differential equations then emerges. Even more recently an arithmetic analogue of the partial differential equations relevant to Riemannian geometry, gauge theory, and relativity has been developed. The talk will be an introduction to this circle of ideas, with ample historical motivation, and numerous examples.