Title: The structure of Gorenstein-linear resolutions of Artinian algebras

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Abstract: Let $\boldsymbol{k}$ be a field, A a standard-graded Artinian Gorenstein $\boldsymbol{k}$-algebra, S the standard-graded polynomial ring $\operatorname{Sym}_\bullet^{\boldsymbol{k}} A_1$, I the kernel of the natural map $S \twoheadrightarrow A$, d the vector space dimension $\dim_{\boldsymbol{k}} A_1$, and n the least index with $I_n \neq 0$. Assume that $3 \leq d$ and $2 \leq n$. We give the structure of the minimal homogeneous resolution $\mathbf{B}$ of A by free S-modules, provided $\mathbf{B}$ is Gorenstein-linear. (Keep in mind that if A has even socle degree and is generic, then A has a Gorenstein-linear minimal resolution.)

Our description of $\mathbf{B}$ depends on a fixed decomposition of A_1 of the form $\boldsymbol{k}x_1 \oplus V_0$, for some non-zero element x_1 and some $(d-1)$ dimensional subspace V_0 of A_1. Much information about $\mathbf{B}$ is already contained in the complex $\overline{\mathbf{B}} = \mathbf{B}/x_1\mathbf{B}$, which we call the skeleton of $\mathbf{B}$. One striking feature of $\mathbf{B}$ is the fact that the skeleton of $\mathbf{B}$ is completely determined by the data (d,n); no other information about A is used in the construction of $\overline{\mathbf{B}}$.

The skeleton $\overline{\mathbf{B}}$ is the mapping cone of $\text{zero} : \mathbb{K} \to \mathbb{L}$, where $\mathbb{L}$ is a well known resolution of Buchsbaum and Eisenbud; $\mathbb{K}$ is the dual of $\mathbb{L}$; and $\mathbb{L}$ and $\mathbb{K}$ are comprised of Schur and Weyl modules associated to hooks, respectively. The decomposition of $\overline{\mathbf{B}}$ into Schur and Weyl modules lifts to a decomposition of $\mathbf{B}$; furthermore, $\mathbf{B}$ inherits the natural self-duality of $\overline{\mathbf{B}}$.

The differentials of $\mathbf{B}$ are explicitly given, in a polynomial manner, in terms of the coefficients of a Macaulay inverse system for A. In light of the properties of $\overline{\mathbf{B}}$, the description of the differentials of $\mathbf{B}$ amounts to giving a minimal generating set of I, and, for the interior differentials, giving the coefficients of x_1. As an application we observe that every non-zero element of A_1 is a weak Lefschetz element for A.

This talk is about joint work with Sabine El Khoury.